

Measurement of Residence Time, Air Entrainment Rate, and Base Pressure in the Near Wake of a Cylindrical Body in Supersonic Flow

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Residence time was measured of species in the near wake behind a flat-faced cylindrical body aligned with the flow in a low-speed wind tunnel. The air entrainment rate into the near wake was calculated from the residence time. The residence time was measured by injecting carbon dioxide (CO₂) into the near wake and then monitoring the time required for dissipation of the gas after the injection was shut off. The dissipation was monitored using a radiometer that measured absorption of radiation from an infrared (IR) source in the 4.3- μ m CO₂ band. The test body was 2½ in. in diameter and 5 in. long. Residence time varied from 14.4 ms at an air velocity of 150 ft/s to 8.98 ms at 312 ft/s. The base pressure on the downstream face of the body was measured and was in reasonable agreement with values in the literature.

Introduction

THIS paper describes an experimental investigation of flow residence times in the recirculation zone or near wake of a circular cylinder axially aligned with a freestream flow as shown in Fig. 1. The data are of direct importance to fluid dynamicists who must design systems where entrainment into wakes is important. A limited amount of work is available in the literature that describes theoretical or experimental flow, residence times, and base pressures in the circulation zones behind bodies immersed in subsonic flows.¹⁻¹⁰

Residence time was measured by injecting a small amount of CO₂ into the near wake behind the test body and then monitoring the time required for it to dissipate after being shut off. The dissipation was monitored with a filtered radiometer that measured the absorption of radiation from an IR source in the 4.3- μ m CO₂ band. Measurements also were made of the pressure on the downstream face of the body, a parameter that affects aerodynamic drag. The experimental method should be useful for a number of different applications including noncylindrical shapes and supersonic flows.

Stirred Reactor Model of the Near Wake

Because the near wake is a complicated region to analyze in detail, a simplified model was used that treated the near wake as a perfectly stirred reactor. This assumed that the region is a homogeneous volume where freestream air and/or other gases are input and mixed products leave to form the output.

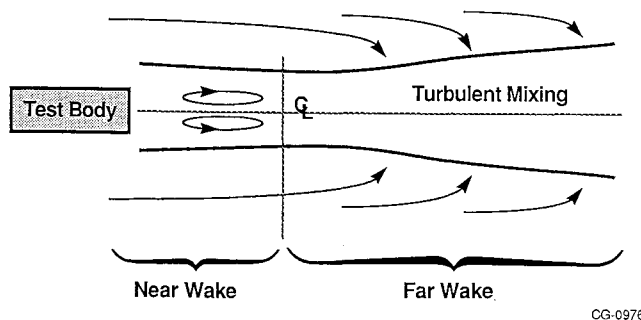


Fig. 1 Schematic of wake behind cylindrical test body.

Consider that the region contains M grams of substance. There are m_e g/s entering and m_o g/s in the output, and

$$\frac{dM}{dt} = m_e - m_o \quad (1)$$

The rate of increase in the region of a given chemical species is

$$\frac{d}{dt}(\alpha M) = \alpha \frac{dM}{dt} + M \frac{d\alpha}{dt} = \alpha_e m_e + \dot{\alpha}_g M - \alpha_o m_o \quad (2)$$

where α is the mass fraction (concentration) of the species in the region, α_e is the mass fraction entering, $\dot{\alpha}_g$ is the rate of production of the species in the region caused by a physical or chemical reaction, and α_o is the mass fraction leaving. It should further be clear that $\alpha_o = \alpha$. Combining Eqs. (1) and (2) and using $\alpha_o = \alpha$ gives

$$\frac{d\alpha}{dt} = \frac{m_e}{M}(\alpha_e - \alpha) + \dot{\alpha}_g \quad (3)$$

If the production factor $\dot{\alpha}_g$ is zero, Eq. (3) becomes

$$\frac{d\alpha}{dt} + \frac{1}{\tau} \alpha = \frac{1}{\tau} \alpha_e \quad (4)$$

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where $\tau = M/m_e$. With τ constant, the solution of this differential equation is

$$\alpha = \alpha_e + C_o e^{-t/\tau} \quad (5)$$

where C_o represents an initial concentration in excess of the input concentration α_e . In the case under consideration here, C_o is due to an injection of CO_2 that is then shut off, and α_e is the mass fraction of CO_2 in the tunnel freestream air. The excess concentration decreases exponentially with a time constant τ , and the concentration α in the region approaches the concentration α_e in the input.

Air Entrainment and Residence Time

The factor M/m_e in Eq. (4), given by the symbol τ , represents a "filling" time or residence time for the reactor; i.e., if mass is being taken from this volume and replenished at an input rate of m_e g/s, then $\tau = M/m_e$ seconds are required to remove and replace the quantity M . The mass M is given by $\rho_i V$, where ρ_i is the total mass density in the near wake and V is the near-wake volume. Since in the case investigated here the freestream air entrainment is more than an order of magnitude greater than the CO_2 injection rate, the density ρ_i is approximately equal to the freestream air density ρ_∞ . The molecular weight of the flow in the wake is not affected by the small amount of excess CO_2 .

The near-wake volume V was assumed to have a cylindrical shape with a diameter equal to the body diameter and a length equal to one body diameter, giving a volume $= 2\pi r^3$ where r is the body radius. Although this was a somewhat arbitrary assumption, a detailed study of the near wake based on solutions obtained with the Navier-Stokes equations for this region concluded that this is a good assumption for the volume, at least for blunt, flat-faced cylinders.

The quantity m_e in this case is the mass rate of freestream air entrained into the near wake. Let the air entrainment rate m_e be a certain proportion K of the air that would flow through the frontal area of the body at the freestream velocity:

$$m_e = K \rho_\infty u_\infty \pi r^2 \quad (6)$$

where ρ_∞ and u_∞ are the freestream air density and velocity and T is temperature. The residence time is, then, with $\rho_i \approx \rho_\infty (T_{\text{static}}/T_{\text{total}})$,

$$\tau = \frac{M}{m_e} = \frac{\rho_i V}{m_e} \approx \frac{\rho_\infty (2\pi r^3)}{K \rho_\infty u_\infty \pi r^2 (T_{\text{total}}/T_{\text{static}})} \quad (7)$$

$$\approx \frac{2r}{K u_\infty (T_{\text{total}}/T_{\text{static}})} \quad (8)$$

In this work, τ was experimentally determined from Eq. (5), and the value of K was then determined from Eq. (8).

Description of Experiment

A cylindrical test body approximately 5 in. long and 2.5 in. in diameter was mounted in a low-speed wind tunnel as shown in Fig. 2. The wind-tunnel test section was 20 in. in diameter and could produce air velocities from 140 to 320 ft/s. CO_2 was injected through a nozzle in the base into the near wake at a rate that would not disturb the base flow. When the CO_2 injection was shut off, the CO_2 concentration decreased approximately exponentially with time constant τ in accordance with Eq. (5) toward the freestream concentration.

Analysis of the time required for the CO_2 valve to close shows that approximately 2 ms were required. The valve is located at the nozzle opening so that there is no delay caused by residual CO_2 remaining in the plumbing downstream of the valve. The valve stem weighs 34 g and is driven by a spring

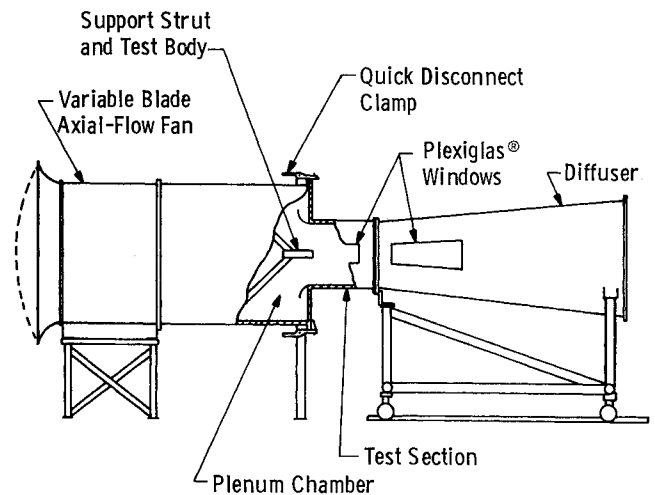


Fig. 2 Ground-level low-speed wind-tunnel facility.

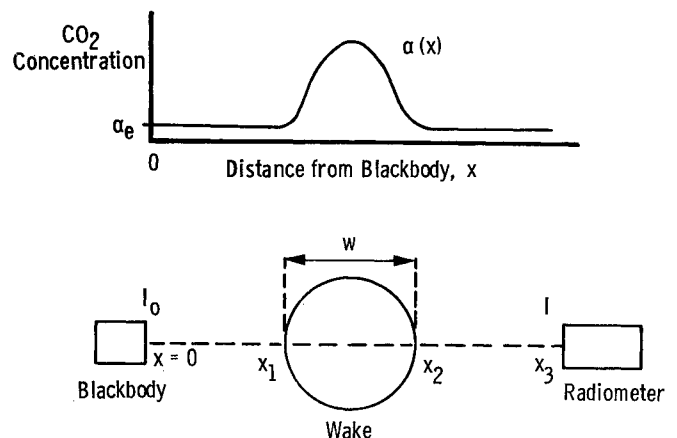


Fig. 3 Physical arrangement of absorption apparatus (bottom) and corresponding CO_2 concentration (top).

tension of 414 g so that the stem accelerates at about 12.18 g while closing. Calibration of the flow through the valve shows that the stem moves a total of 0.0765 in. to close, but 95% of the flow is closed off in the final 0.044 in. of travel. If a constant acceleration of 12.18 g is assumed for the stem, the final 0.044 in. of travel to shut off the CO_2 requires 1.99 ms. The question of whether this finite valve closing time has a significant influence on the subsequent decay time determination is discussed later.

The CO_2 concentration was monitored by an IR absorption apparatus where a heated blackbody source was placed on one side of the wake as shown in Fig. 3. An IR radiometer, filtered to pass radiation in the 4.3- μm -wavelength CO_2 band, was placed on the other side of the wake to measure the source radiation, which was attenuated by the presence of the CO_2 . The test body could be traversed fore and aft axially along the tunnel centerline, permitting absorption measurements to be made at a distance varying from 1.5 to 6 in. downstream of the base. Also shown in Fig. 3 is an assumed CO_2 concentration profile as a function of the distance x from the blackbody source to the radiometer. The CO_2 concentration is at its freestream atmospheric value (approximately 0.03% by mole) along the path outside the wake and within the wake has a profile in excess of the freestream value.

Monitoring CO_2 Concentration

Without CO_2 injection into the near wake, the intensity I_1 measured by the radiometer was simply the source intensity I_0

attenuated by the atmospheric CO₂ concentration over the distance x_3 :

$$I_1 = I_0 \exp\{-\sigma \alpha_e \rho x_3\} \quad (9)$$

where σ is the attenuation cross section in cm² per gram of CO₂, α_e is the atmospheric CO₂ concentration (mass fraction), and ρ is the atmospheric density in g/cm³. With CO₂ injected into the wake, the measured intensity I is

$$I = I_0 \exp\left\{-\sigma \alpha_e \rho (x_3 - x_2 + x_1) - \sigma \rho \int_{x_1}^{x_2} \alpha(x) dx\right\} \quad (10)$$

where x_1 , x_2 , and x_3 are identified in Fig. 3 and $\alpha(x)$ is the CO₂ concentration profile along x , and it is again assumed that the mass density in the near wake is the same as that outside the wake.

Defining the average concentration α_{av} in the wake as

$$\alpha_{av} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \alpha(x) dx \quad (11)$$

Eq. (10) can be written

$$I = I_0 \exp\{-\sigma \alpha_e \rho (x_3 - x_2 + x_1) - \sigma \alpha_{av} \rho (x_2 - x_1)\} \quad (12)$$

Identifying the average wake concentration α_{av} with the concentration α of Eq. (5) gives

$$\begin{aligned} I &= I_0 \exp\{-\sigma \alpha_e \rho (x_3 - x_2 + x_1) - \sigma \rho (\alpha_e + C_o e^{-t/\tau})(x_2 - x_1)\} \\ &= I_0 \exp\{-\sigma \alpha_e \rho x_3 - \sigma \rho C_o e^{-t/\tau} (x_2 - x_1)\} \end{aligned} \quad (13)$$

Dividing Eq. (13) by Eq. (9) and letting the wake diameter $(x_2 - x_1) = w$ finally gives

$$\frac{I}{I_1} = \exp\{-\sigma w \rho C_o e^{-t/\tau}\} \quad (14)$$

$$\ln\left(\frac{I}{I_1}\right) = -\sigma w \rho C_o e^{-t/\tau} \quad (15)$$

This means that the natural log of the intensity measured by the radiometer can be plotted as a function of time when the CO₂ injection is shutt off and an exponential can be fitted through this plot to determine the time constant τ .

Experimental Results

Figure 4 is an example of a plot of the log of the measured intensity I vs time. The figure represents a case where the tunnel air velocity was 146 ft/s and the absorption measure-

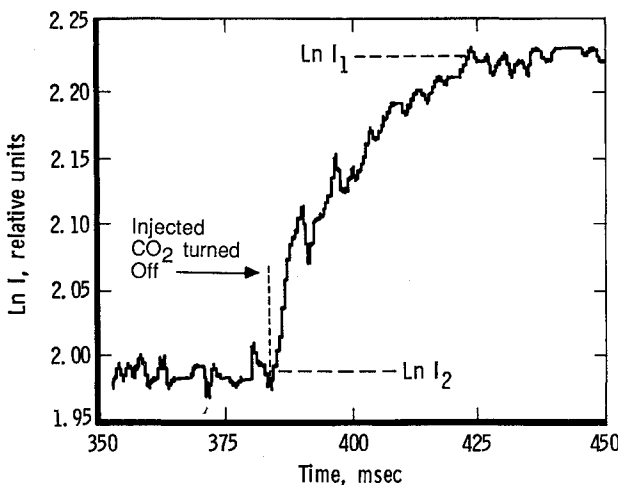


Fig. 4 Logarithm of measured intensity vs time.

ment was made 1.5 in. downstream of the test body. The left side of the plot where $\ln I = \ln I_2$ represents the time when the CO₂ injection was taking place and the maximum CO₂ concentration in the wake was absorbing the radiation.

When the CO₂ injection was turned off, the CO₂ was swept out of the wake by the free airstream and the measured intensity increased to the value I_1 , where the only absorption was due to the natural CO₂ in the atmosphere. From Eq. (15) at $t = 0$, the natural log of the ratio of the average intensity before the CO₂ was shut off, I_2 , to the average intensity I_1 after the CO₂ had dissipated is equal to the product $\sigma w \rho C_o$:

$$\ln\left(\frac{I_2}{I_1}\right) = -\sigma w \rho C_o \quad (16)$$

where σ is the attenuation cross section in cm² per gram of CO₂, w is the wake width, ρ is the atmospheric mass density, and C_o is the excess CO₂ mass fraction at and before the time $t = 0$ when the CO₂ injection was turned off.

For the case shown in Fig. 4,

$$\sigma w \rho C_o = -\ln\left(\frac{I_2}{I_1}\right) = \ln I_1 - \ln I_2 = 0.244 \quad (17)$$

Because the cross section σ depends in a complicated way on wavelength and temperature, no effort was made to calculate a value for the absolute concentration C_o .

Residence Time

Figure 5 is the data of Fig. 4 prepared for curve fitting for determination of the decay time constant τ . The difference $(\ln I_1 - \ln I)$ is plotted vs $(t - t_s)$, where the start time t_s for this plot has been selected from Fig. 4 to be $t_s = 382$ ms. In effect, Fig. 4 has been turned upside down. This plot has been fitted using linear regression with the curve

$$(\ln I_1 - \ln I) = A e^{-b(t - t_s)} \quad (18)$$

where A and b are constants to be determined and, from Eq. (15), $b = 1/\tau$. In this case, the residence time was found to be $\tau = 13.8$ ms. Table 1 lists test conditions with the concentration factors $(\sigma w \rho C_o) = -\ln(I_2/I_1)$ and the residence times τ .

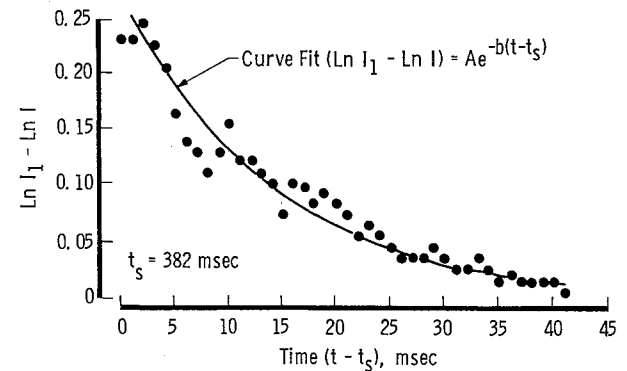


Fig. 5 Data of Fig. 4 with curve-fit to determine residence time.

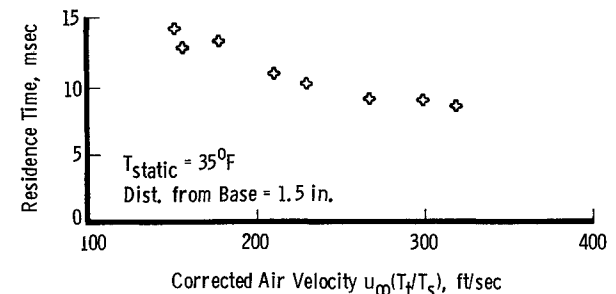


Fig. 6 Residence time vs product of air velocity and temperature correction factor.

Table 1 Test conditions and results

Data point number	Distance downstream from can, in.	Tunnel air velocity, ft/s	CO ₂ injection pressure, psig	$\ell n(I_1/I_2) = \sigma w \rho C_o$	Residence time τ , ms
1	1.5	144	13.3	0.2690	11.28
2		146	13.3	0.2444	13.80
3		150	5.8	0.2268	14.40
4		151	3.9	0.1883	14.21
6		155	5.0	0.2321	13.21
7		177	5.1	0.2237	13.52
8		209	5.3	0.1790	11.16
9		226		0.1349	10.27
11		264		0.1162	9.19
12		294	5.3	0.1017	9.13
13	1.5	312	5.1	0.09149	8.98
21	3.0	156	4.7	0.07740	11.95
22	3.0	157	26.8	0.1391	12.99
23	3.0	318	27.6	0.09599	11.02
24	4.0	156	27.5	0.1369	11.91
25	4.0	319		0.08355	9.66
26	6.0	156		0.1085	9.30
27		315	27.5	0.05680	6.98
28		316	54.9	0.09245	12.60
29	6.0	317	5.6	0.02861	9.67

Air Entrainment Rate

As discussed earlier, the air entrained into the near wake was assumed to be a proportion K of the air that would pass through the frontal area of the cylindrical test body, where the value of K is to be determined. From Eq. (8), the residence time is inversely proportional to the freestream air velocity u_∞ , modified by a temperature correction factor ($T_{\text{total}}/T_{\text{static}}$) as shown plotted in Fig. 6. The temperature correction factor was small, varying from 1.004 to a maximum of 1.016.

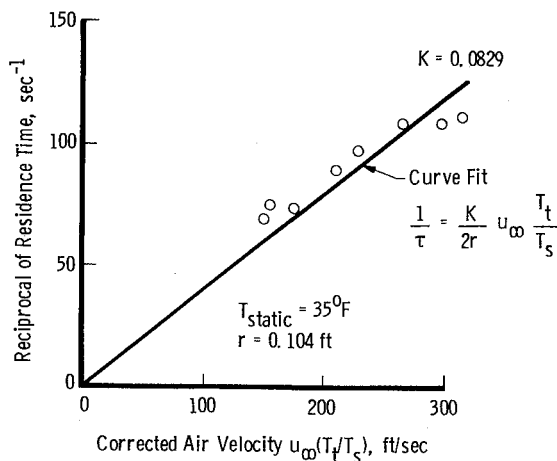


Fig. 7 Reciprocal of residence time vs product of air velocity and temperature correction factor.

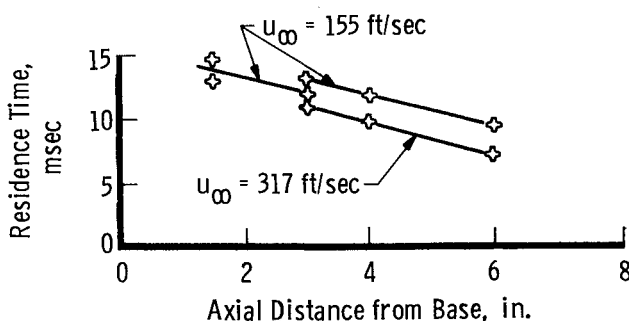


Fig. 8 Residence time vs axial distance downstream from test body.

To determine the value of K , the reciprocal of the residence time was plotted vs the product $u_\infty(T_{\text{total}}/T_{\text{static}})$, and K was calculated from the slope of the best-fit straight line through the data. The plot is shown in Fig. 7, and the value of K is 0.08; i.e., 8% of the air that would pass through the frontal area of the test body is entrained. The measured residence time is plotted vs axial distance behind the body for two different tunnel velocities in Fig. 8.

Initial CO₂ Concentration

The concentration factor ($\sigma w \rho C_o$) is plotted vs tunnel freestream air velocity in Fig. 9 for constant CO₂ injection flow rate and a position 1.5 in. downstream of the test body. Since the attenuation cross section σ , wake width w , and total mass density ρ should all remain the same with varying air velocity at this axial position, the plot simply indicates that

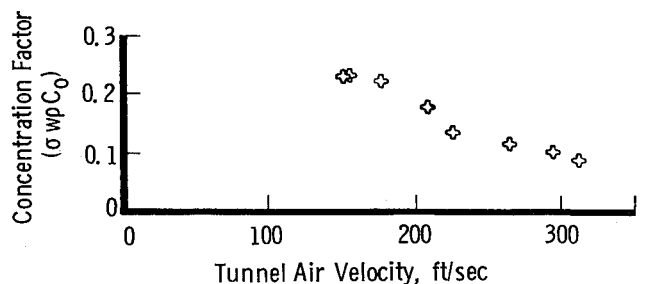


Fig. 9 Concentration factor ($\sigma w \rho C_o$) vs air velocity.

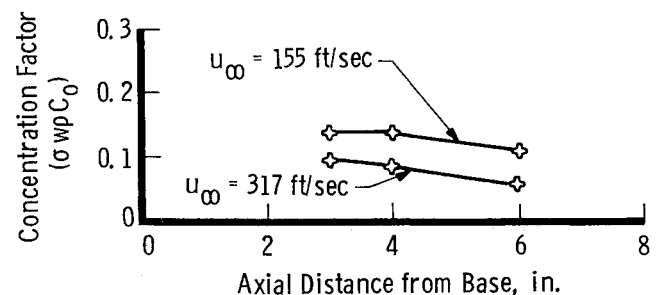


Fig. 10 Concentration factor ($\sigma w \rho C_o$) vs axial distance downstream from test body.

Table 2 Base pressure coefficient

Data point number	Tunnel air velocity, ft/s	Mach no., M_∞	Tunnel wall pressure p_∞ , psi	Wall base pressure, $(p_\infty - p_b)$, psi	Base pressure coefficient
15	156	0.143	14.0	0.0125	-0.0624
16	204	0.187	13.9	0.0195	-0.0802
17	246	0.225	13.8	0.0294	-0.0601
18	301	0.276	13.6	0.0466	-0.0643
19	319	0.292	13.5	0.0680	-0.0844

the increasing air velocity results in lower initial excess CO_2 concentration C_o .

The concentration factor is plotted vs axial position in Fig. 10. Here the wake width w cannot be assumed to remain constant with distance, so the specific concentration variation is unknown.

Base Pressure

An additional parameter of interest is the base pressure p_b on the downstream end of the test body. The gas injection nozzle opening was refitted to be used as a port to measure the pressure under conditions of different tunnel air velocities. Table 2 gives the measured base pressure coefficients C_{pb} for several air velocities. The base pressure coefficient is given by

$$C_{pb} = \frac{-2(p_\infty - p_b)}{\rho_\infty \gamma M_\infty^2} \quad (19)$$

where p_∞ and M_∞ are the freestream pressure and Mach number, p_b is the base pressure, and γ is the ratio of air specific heats (taken to be 1.4).

The values of the base pressure coefficient are in reasonable agreement with experimental data from other sources.^{1-3,10,11} The base pressure coefficient remains fairly constant vs Mach number for subsonic Mach numbers up to 0.8 and for a given geometry, but this value may vary from roughly -0.05 to -0.16, depending on the character of the boundary layer ahead of the base. See Fig. 11 for details.

Effect of CO_2 Injection Valve Closing Time

Equation (4) can be modified to include the effect of the finite closing time of the CO_2 injection valve. Let the input to the stirred reactor before closing be $\beta + m_e$, where β is the CO_2 nozzle flow rate in g/s, and m_e is the total number of g/s entrained from the airstream. The concentration (mass fraction) of CO_2 in the input is

$$\frac{\beta + \alpha_e m_e}{\beta + m_e}$$

where α_e is the concentration in the airstream.

Furthermore, assume that $m_e \gg \beta$. During calibration of the nozzle with air, the flow rate through the nozzle under a pressure of 50 psig was measured to be 2.35 SCFM, or about 1.362 g/s. In contrast, the air entrainment rate m_e is an order of magnitude greater, being about 16.34 g/s at an airstream velocity of 150 ft/s. Therefore, the input CO_2 concentration is approximately

$$\frac{\beta + \alpha_e m_e}{m_e} = C + \alpha_e \quad (20)$$

where $C = \beta/m_e$ is the concentration in excess of atmospheric concentration.

Next let the valve close in 2 ms such that the input concentration is given by

$$C(1 - at) + \alpha_e \quad (0 \leq t \leq 1/a)$$

where $1/a$ is 0.002 s. Equation (4) can now be written for the time during the closing of the valve as

$$\frac{d\alpha}{dt} + \frac{1}{\tau} \alpha = \frac{1}{\tau} \{C(1 - at) + \alpha_e\} \quad (0 \leq t \leq 1/a) \quad (21)$$

Solution of this equation gives

$$\alpha = \alpha_e + C\{1 - at + at\tau(1 - e^{-t/\tau})\} \quad (0 \leq t \leq 1/a) \quad (22)$$

After the valve is fully closed, Eq. (5) is applicable:

$$\alpha = \alpha_e + C_o e^{-(t - 1/a)/\tau} \quad (1/a \leq t) \quad (23)$$

where $\alpha_e + C_o$ is the concentration at $t = 1/a$.

Figure 12 is a sample plot of the concentration $(\alpha - \alpha_e)$ vs time calculated using these two equations, for a valve closing time of $1/a = 0.002$ s and a residence time of 0.009 s to examine the effect on residence time determination using the curve-fitting method of this report. The concentration is normalized to $C = 1.0$. In the figure, the calculated concentration is shown sampled at 1-ms intervals, as were the test data. The samples were then curve-fitted using $(\alpha - \alpha_e) = Ae^{-bt}$, which is shown as a dashed line. The residence time determined from the curve fit is $1/b = 0.00908$ s compared to the value 0.009 used for the calculation, an error of less than 1%. In other words, the values of the curve for t less than 2 ms have minimal effect on the curve fit through the rest of the data, and it can be concluded that the finite valve closing time did not significantly affect the residence time determination.

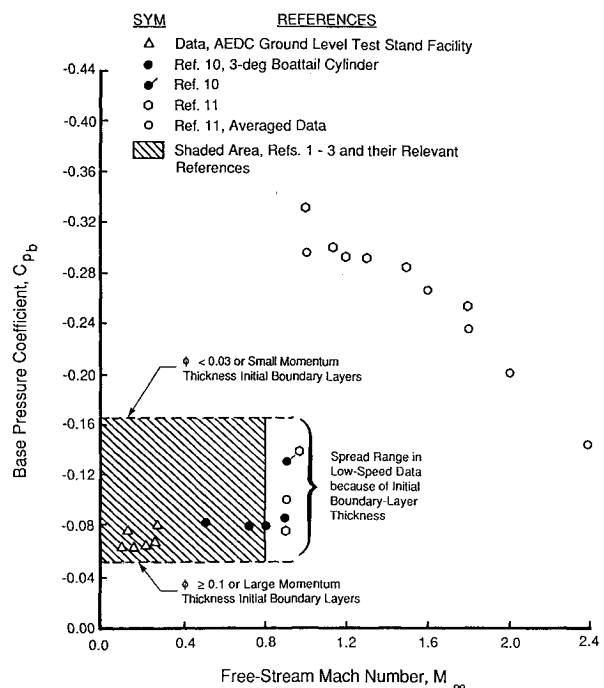


Fig. 11 Base pressure coefficient of circular cylinder bluffbody vs Freestream Mach number.

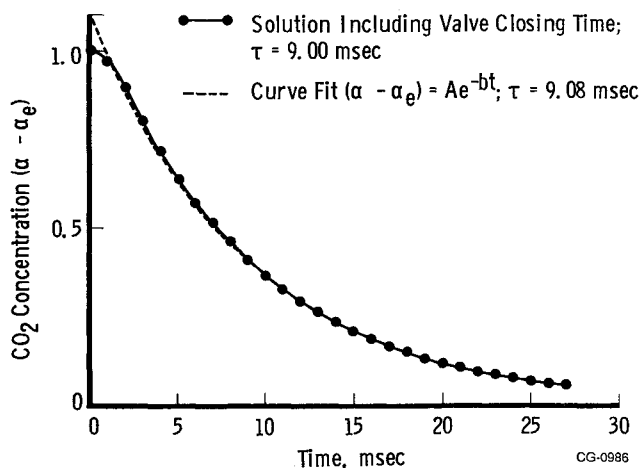


Fig. 12 Calculated CO_2 concentration decay including effect of CO_2 valve closing time.

Summary

The residence time in the near wake behind a flat-faced cylindrical body was measured in a low-speed wind tunnel for various freestream air velocities. The residence time is a measure of the time required for the recirculating air in the near wake to be replaced by fresh air entrained into the wake and is inversely proportional to the rate of entrainment. The residence time was measured by injecting CO_2 into the near wake and monitoring its dissipation after the CO_2 was shut off by use of a radiometer that measured absorption of IR radiation by the CO_2 . The residence time varied from 14.4 ms at an air velocity of 150 ft/s to 8.98 ms at 312 ft/s. Plotting the reciprocal of the residence time vs air velocity determined a value for the entrainment constant $K = 0.08$; i.e., 8% of the free airstream that would pass through the frontal area of the test body is drawn into the near wake.

Finally, the pressure on the downstream face or base of the test body (a parameter related to the aerodynamic drag) was measured for several air velocities. The results were comparable to other values reported in the literature.

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